Performance analysis of IMC based PID controller tuning on approximated process model

Ankit K. Shah, Markana Anilkumar and Nishant Parikh

***Abstract*—Classical Proportional Integral Derivative(PID) controller remains the most popular approach for industrial process control. Poor tuning of PID controller can lead to mechanical wear associated with excessive control activity, poor control performance and even poor quality products. In this paper, we design procedure for the internal model control(IMC) approach for tuning of conventional PID controller with proper tuning rules. Furthermore, with help of analytical rule of step test obtaining the effective first order time delay model of the process. A simulation example of continuous stirred tank reactor is used in which the IMC based PID tuning method implemented and the step response of the closed loop system is compared with classical tuning methods like Ziegler-Nichols and Cohen-Coon.**

***Index Terms*—Normalized cutset, discrete wavelet transform, high boost filter**

I. INTRODUCTION

NEVERTHELESS , PID controllers are still widely used in industrial applications including for process control. The reason is that PID controller has a simple structure which is easy to be understood by the engineers who design it. This is not only due to the simple structure which is conceptually easy to understand and, which makes manual tuning possible, but also to the fact that the algorithm provides adequate performance in the vast majority of applications. It is widely used in process industries because of its simple structure and robustness to the modeling error. Sophisticated control algorithms, such as model predictive control, are built on the basis of the PID algorithm. Even in non-linear control development, PID control has been used as comparison reference [1].

According to a survey conducted by Japan electric Measuring Instruments Manufacturers Association in 1989, 90 percent of the control loops in industries are of PID type [1] and only small portion of the control loop works well. Also survey by ender [2] indicates 30 percent of the controller are operated in manual mode and 20 percent of the loops use factory tuning. It means that PID controller is widely used but poorly tuned. Poor tuning can lead to mechanical wear associated with excessive control activity, poor control performance and even poor quality products. The present work is aimed to provide PID controller tuning guidelines using Internal Model Control(IMC) approach. Recently much research effort has been focused on the automatic tuning of PID controllers, which was first proposed by Astrom and Hagglund (1984) [2]. They have introduced novel relay tuning method for finding the critical gain and critical frequency of closed loop process [2] and proposed several tuning rules for PID controllers based on this information. The PID controller tuning is method of computing the three control parameters Proportional gain, Derivative time and Integral time, such that the controller meets desired performance specification. Since the exact dynamics of the plant is generally unknown, the basic function of autotuners is some experimental procedure through which plant information is obtained in order to compute the controller parameters. Tuning techniques can therefore be classified according to this experimental procedure. This is particularly true for the optimal PID controller tuning for time delay processes since the stability check for a given time-delay closed-loop system is not a trivial task.

In the second section, the design steps for Internal Model Control(IMC) will be describe. In the same section tuning formulas for the conventional PID controller closed loop system will be given. In the third section simulation results on continuous flow stirred-tank reactor(CSTR) will be given with comparison study of closed loop PID controller response using Ziegler-Nichols, Cohen-coon and IMC tuning methods. The fourth section concludes the our approach of IMC based PID controller parameter tuning.

II. BASIC DESIGN OF INTERNAL MODEL CONTROL(IMC)

In this section, we will develop the IMC approach for PID controller tuning. The name comes from the fact that the controller has explicit model of the plant as its part [3]. The premise of IMC is that in reality, we only have an approximation of the actual process. Even if we have the correct model, we may not have accurate measurements of the process parameters. Thus the imperfect model should be factored as part of the controller design.

In the block diagram shown in Fig. 1 [4] implementation of IMC on the process transfer function G_p is given. In that $\tilde{G}_p$ is the approximate transfer function of the process G_p and $\bar{G}_c$ is the model controller. In Fig. 2 block diagram of the conventional feedback controller shown. By comparison of Fig. 1 and Fig. 2, conventional feedback controller G_c consists of $\bar{G}_c$ and $\tilde{G}_p$. We first need to derive the closed-loop functions

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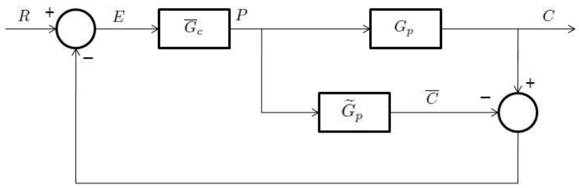

Fig. 1. Block diagram of internal model control structure

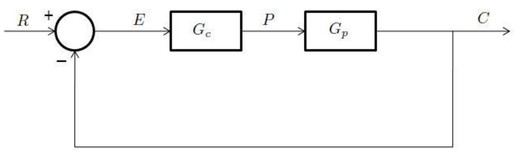

Fig. 2. Block diagram of conventional PID structure

for the IMC system. Based on the block diagram of the IMC, the error is E = R-(C -$\bar{C}$) where $\bar{C}$ = $\tilde{G}_p$. We can rearranged the equations and get the controller output P as

$$P = \frac{\overline{G}_c}{1 - \overline{G}_c \tilde{G}_p}(R - C) \tag{1}$$

This step is to shows the relationship between the conventional feedback controller transfer function G_c shown in Fig. 2 and IMC structure in Fig. 1:

$$G_c = \frac{\overline{G}_c}{1 - \overline{G}_c \tilde{G}_p} \tag{2}$$

The poles of G_c are inherited from the zeros of G_p. If G_p has positive zeros, it will lead to a G_c function with positive poles. To avoid that, we split the approximate function as a product of two parts:

$$\tilde{G}_p = \tilde{G}_{p+} + \tilde{G}_{p-} \tag{3}$$

with $\tilde{G}_{p+}$ containing all the positive zeros, if present. The controller will be designed on the basis of $\tilde{G}_{p-}$.. only. We now define the model controller transfer function $\bar{G}_c$ as

$$\overline{G}_c = \frac{G_f}{\tilde{G}_{p-}} \tag{4}$$

where G_f is first order low pass filter [5] used to avoid model mismatch. The filter transfer function G_f defined as

$$G_f = \frac{1}{\tau_f s + 1} \tag{5}$$

where τ_f is the filter time constant.

III. PID TUNING USING IMC

For conventional PID controller transfer function is given by

$$G_c(s) = K_p[1 + \frac{1}{\tau_i s} + \tau_d s] \tag{6}$$

where K_p is the proportional gain, τ_i is the integral time and τ_d is the derivative time or lead time.
To model our process by fitting the open-loop step test data as a first order function with time delay, our measured or approximate model transfer function $\tilde{G}_p$ given by

$$\tilde{G}_p = \frac{K e^{-t_d s}}{\tau_p s + 1} \tag{7}$$

where K represents the dc gain, τ_p and t_d the process time constant and time delay, respectively. Replacing the time delay of by a first order Pade rational approximation expressed by [5]

$$e^{-t_d s} \approx \frac{1 - \frac{t_d}{2}s}{1 + \frac{t_d}{2}s} \tag{8}$$

The approximate model transfer function represent in (7) can be factorized in to invertible and noninvertible factors as

$$\tilde{G}_{p-} = \frac{K}{(\tau_p s + 1)(1 + \frac{td}{2}s)} \tag{9}$$

$$\tilde{G}_{p+} = (1 - \frac{t_d}{2}s) \tag{10}$$

From (9) and (4), IMC controller transfer function can be derive as

$$\overline{G}_c = \frac{(\tau_p s + 1)(1 + \frac{t_d}{2}s)}{K(\tau_f s + 1)} \tag{11}$$

Comparing equation (11) with the ideal PID controller equation represented by (6), which will lead to the tuning parameters of an ideal PID controller based on IMC approach. Controller parameters are given as

$$K_p = \frac{2\frac{\tau_p}{t_d} + 1}{K(2\frac{\tau_f}{t_d} + 1)} \tag{12}$$

$$\tau_i = \frac{t_d}{2} + \tau_p \tag{13}$$

$$\tau_d = \frac{\tau_p}{2\frac{\tau_p}{t_d} + 1} \tag{14}$$

IV. SIMULATION RESULTS ON CONTINUOUS STIRRED TANK REACTOR(CSTR)

Fig. 3 shows the continuous stirred tank reactor for reactant A. In this example, we have a stirred tank with a volume V of $4m^3$ being operated with an inlet continuous flow rate Q of $0.4m^3/sec$ and which contains an inlet reactant A at a concentration C_{in}.

The model equation for continuous flow stirred-tank reactor with chemical reaction of the reactant A given as

$$V\frac{d}{dt}C_A = Q(C_{in} - C_A) - V\lambda C_A \tag{15}$$

where C_A is the molar concentration of reactant A and λ is the first order reaction rate constant of value $0.1sec^{-1}$. Above equation can be written in simplified manner by

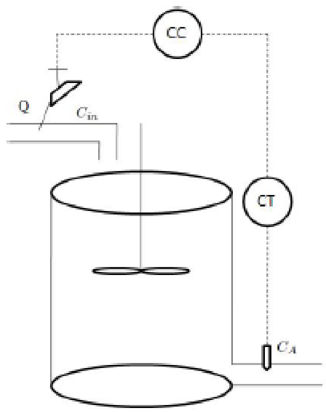

Fig. 3. Continuous stirred tank reactor

$$\frac{V}{Q}\frac{d}{dt}C_A + (1 + \frac{KV}{Q})C_A = C_{in} \tag{16}$$

By taking laplace transformation of above equation the transfer function of CSTR can be written as

$$G_{ps} = \frac{C_A}{C_{in}} = \frac{\frac{1}{1+\frac{KV}{Q}}}{\frac{V}{Q+KV}s + 1} \tag{17}$$

The analog output of the concentration detector is transmitted to a controller, which in turn sends a signal to the injection regulating valve at input stream. Photodetector is used to monitor the concentration of reactant A. The magic photodetector is extremely fast and the response is linear over a large concentration range. Unite of concentration C_A in terms of $gmol/m^3$ now it converted into mV using the measurement gain $2.6mVm^3/gmol$ and transport lag t_d = 0.7sec. The regulating valve is especially designed so that inlet concentration of the reactant A in $gmol/m^3$ varies linearly with the valve position. The regulating valve is thus first order with a time constant of 0.2 sec and a steady state gain of 0.6 $gmol/m^3mV$. Now complete system open loop block diagram show in Fig. 4, in that G_m is the measurement transfer function, G_a is the valve transfer function and G_{ps} is the actual transfer function. Transfer function of G_m , G_a and G_{ps} are given by

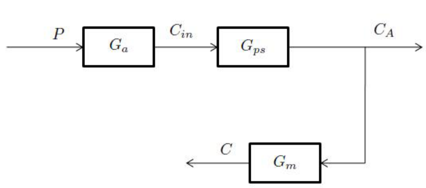

Fig. 4. Complete open loop block diagram of CSTR

$$G_{ps} = \frac{0.67}{6.67s + 1} \tag{18}$$

TABLE I
PID CONTROLLER PARAMETERS TUNING RULES

Parameters	Ziegler-Nichols	Cohen-Coon	IMC
K_p	$\frac{1.2\tau_p}{K t_d}$	$\frac{1}{K}\left(\frac{4\tau_p}{3t_d} + \frac{1}{4}\right)$	$\frac{2\frac{\tau_p}{t_d}+1}{K\left(2\frac{\tau_f}{t_d}+1\right)}$
τ_i	$2t_d$	$t_d\frac{32+6(\frac{t_d}{\tau_p})}{13+8(\frac{t_d}{\tau_p})}$	$\frac{t_d}{2} + \tau_p$
τ_d	$0.5t_d$	$t_d\frac{4}{11+2(\frac{t_d}{\tau_p})}$	$\frac{\tau_p}{2\frac{\tau_p}{t_d}+1}$

TABLE II
VALUES OF PID CONTROLLER PARAMETERS

Parameters	Ziegler-Nichols	Cohen-Coon	IMC
K_p	9.64	10.95	6.87
τ_i	1.7	2	7.52
τ_d	0.43	0.3	0.4

$$G_m = 2.6e^{-0.7s} \tag{19}$$

$$G_a = \frac{0.6}{0.2s + 1} \tag{20}$$

Complete open loop transfer function of the CSTR given as

$$G_p = \frac{(0.67)(0.6)(2.6)e^{0.7s}}{(6.67s + 1)(0.2s + 1)} \tag{21}$$

However, to use the empirical tuning relations, we need to fit the data to a first order transfer function with dead time. Thus at this stage, we probably would have obtained the approximation of the CSTR by giving step signal to the input injection regulating valve. From the Fig. 5, first order approximate transfer function of complete CSTR system is given by

$$G_p = \frac{1.04e^{-0.85}}{7.1s + 1} \tag{22}$$

From equation (12), (13), (14) and (22) values of PID controller parameters are

$$K_p = 6.87, \quad \tau_i = 7.52sec, \quad \tau_d = 0.4sec \tag{23}$$

For the Ziegler-Nichols [6] and Cohen-Coon [7] methods conventional PID parameters are evaluated using tuning rules given in Table 1. The values of PID controller parameters for IMC, Ziegler-Nichols and Cohen-Coon tuning rules are given in Table 2. From Table 2, IMC tune PID controller have proportional gain less and integral time more as compare to other two methods. The unit step response for the close loop control of CSTR shown in Fig. 6 using above setting

TABLE III
CLOSE LOOP RESPONSE PARAMETERS USING DIFFERENT TUNING METHODS

Tuning Method	Peak Overshoot(percentage)	Settling time(sec)
Ziegler-Nichols	87	8.7
Cohen-Coon	105	15.8
IMC	13	5.2

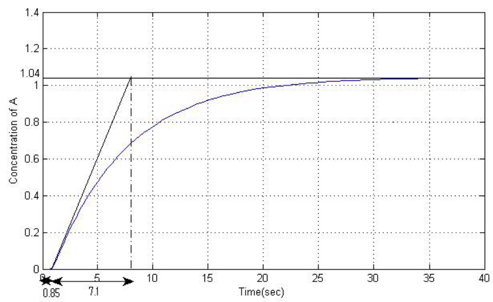

Fig. 5. Open loop step response of CSTR Structure

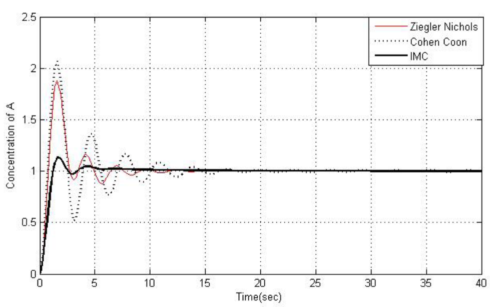

Fig. 6. Closed loop response using different PID tuning methods

of PID controller parameters for given three methods. The comparison between the Ziegler-Nichols, Cohen-Coon and IMC PID tuning rules closed loop step response given in Table 3.

V. CONCLUSION

Internal model control methodology elementary notions were reviewed, with particular relevance being given to the conversion from the IMC structure to a conventional PID controller configuration. From these we can able tune the gains PID controller by using IMC parameters. From Fig. 6 and Table 3, IMC based PID controller unit step response required less settling time to reach desired concentration and give less overshoot in the response compared to other two methods. We can conclude that the IMC based tuning of PID controller outperformed the Ziegler-Nichols and Cohen-Coon tuning methods.

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